

ACCIDENT PREDICTION MODELS FOR SIGNALISED INTERSECTIONS

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ABSTRACT

Beca has developed a number of Accident Prediction Models (APM) for accidents at signalised intersections. This paper presents and discusses APMs that have been developed for signalised intersections in New Zealand. Flow-only models are now available for reported motor-vehicle, and cyclist and pedestrian versus motor-vehicle accidents. Research has now considered non-flow variables such as intersection geometry and signal phasing and has found that these can be important predictor variables. The models show that for increasing flows the accident rate per vehicle generally decreases, except for rear end motor-vehicle accidents. This effect is largest for accidents involving cyclists where there is a strong 'safety in numbers' effect. Current research into APMs for signals with high approach speeds is also outlined.

INTRODUCTION

Being able to predict the number of accidents at an intersection is a useful tool for transportation professionals. Accident Prediction Models (APMs) enable engineers to evaluate the effect on safety of new intersections, changes in the type of control, impact of remedial measures (such as right turn bays), and the effect of traffic growth. By quantifying these changes an economic evaluation can be prepared, and a safety (or efficiency) project prioritised.

With increasing traffic volumes on our urban roads, particularly in the large cities, existing roundabouts and priority-controlled junctions are being replaced with traffic signals to address capacity, efficiency and safety issues or to provide better amenity for pedestrians and cyclists. As the types and severity of accidents vary between the forms of control, APMs can be used to show the changes in accident costs.

Like a number of other developed countries, APMs have been developed in New Zealand for traffic signals. The majority of models developed internationally have been for accidents involving motor-vehicles only, and for either the major accident types or total accidents. This paper discusses models for traffic signals that have been developed in New Zealand since 1995, and includes the following accident categories:

- Total reported accidents (involving a motor-vehicle);
- Cyclist versus motor vehicle accidents;
- Pedestrians versus motor vehicle accidents; and
- Major accident types;

This paper covers some of the improvements that have been made to the model forms. It concludes with a discussion on current work on developing APMs for high-speed traffic signals.

SIGNALISED INTERSECTION APMS

Turner (2000) developed a series of APMs for a variety of intersection types and different motor-vehicle accident types. Two main types of model were developed for signalised intersections. One

model type relates specific accident types to the turning movements of vehicles involved in such collisions. The second model type relates total accidents at an intersection to the product of two-way traffic volumes on the two intersecting roads. It was this second type of model that was incorporated into Appendix 6 of the Project Evaluation Manual (Transfund, 1997).

Building on previous research, Turner and Roozenburg (2004) added non-flow variables to the models of right-turn-against APMs at 4-arm traffic signals. The objective of this research was to consider a number of non-flow variables, and determine whether these are also key predictors for accident occurrence. The outcome was a more refined accident prediction model for this accident type. Of particular interest was the impact of right-turn phasing on accident occurrence. The key non-flow variables examined were:

- Intersection geometry and layout (e.g. number of through lanes, right turn bay offset and intersection depth);
- Right-turn signal phasing (e.g. filtered turns); and
- Forward visibility to opposing traffic.

A number of the variables included in the models were correlated and hence explain the same variability in the accident observations.

Prior to 2004 no accident prediction models had been developed in New Zealand for the non-motorised modes of travel (walking or cycling). Internationally only a small number of studies have considered these 'active modes'. Turner et. al. (2005) developed models for pedestrian and cyclists accidents involving motor-vehicles at intersections and midblock. These models investigated a small number of non-flow variables in addition to motor vehicle, pedestrian and cycle traffic volumes.

MODEL FORM

The models used in the above studies are called generalised linear models and typically have a negative binomial or Poisson error structure. Generalised linear models were first introduced to road accident studies by Maycock and Hall (1984), and extensively developed in Hauer et al (1989).

The aim of the modelling exercise is to develop relationships between the mean number of accidents (as the dependent variable), and predictor variables, such as traffic, cyclist or pedestrian flows and non-flow variables. Typically the models have the following form:

Equation 1 $A = b_0 x_1^{b_1} x_2^{b_2}$,

where A is the annual mean number of accidents, x_n is the average daily flow of vehicles, pedestrians or cyclists or a continuous non-flow variable, and b_n are the model coefficients. Additional flows or non-flow variables can be added to the model in a multiplicative form by adding various $x_i^{b_i}$ variables on to the end of the equation. In the modelling process, a log-linear transformation is made (refer to Equation 2). This is the reason the models are called linear models even though the final model form is multiplicative.

Equation 2 $\log A = \log b_0 + b_1 \log x_1 + b_2 \log x_2$

Discrete variables, either ordinal or nominal can also be included in the models. Where a variable has two possible values, the models are typically of the form:

Equation 3
$$A = b_0 x_1^{b_1} x_2^{b_2} e^{\pm b_3}$$

In this model form, the two values of $e^{\pm b_3}$ act as multipliers for the ‘flow-only’ model (Equation 1). In the modelling process, the log-linear transform is:

Equation 4
$$\log A = \log b_0 + b_1 \log x_1 + b_2 \log x_2 + b_3$$

Models have either a Poisson or Negative binomial error structure. The ‘Poisson’ model was used where the variance in accident numbers is roughly equal to or less than the mean over the majority of the traffic flow range. However, when the variability is generally higher than the mean the ‘negative binomial’ model is used. The Negative binomial model is a mixture of the Poisson and gamma distributions. The model is described using two parameters k and μ , where k along with the coefficients b_0, b_1, b_2 must be estimated from the data using an iterative process. A more detailed explanation of the models is given in Turner (1995) and Hauer et al (1989).

MODEL INTERPRETATION

There is often confusion regarding interpretation of the relationship between accidents and the variables. This section explains these relationships.

In APMs the parameter b_0 acts as a constant multiplicative value. If the number of reported injury accidents is not dependent on the values of the two predictor variables (x_1 and x_2), then the model parameters b_1 and b_2 are zero. In this situation the value of b_0 is equal to the mean number of accidents. The value of the parameters b_1 and b_2 indicate the relationship that a particular predictor variable has (over its flow range) with accident occurrence. There are five types of relationship, as presented in Figure 1 and discussed in Table 1.

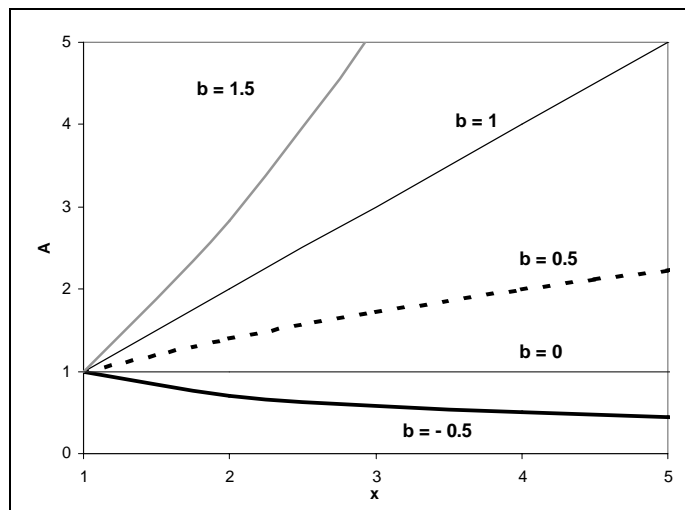


Figure 1: Relationship between accidents and predictor variable x for different values of model exponents (b1)

Generally, accident prediction models have exponents between $b_i = 0$ and $b_i = 1$, with most flow variables having an exponent close to 0.5, i.e. the square root of flow. However in some situations parameters have a value outside this range. For example, rear-end motor vehicle accidents, where

the values of b_i are consistently above 1.0. This is likely to be due to an increase in traffic densities at higher flows which leads to increased interaction between vehicles making rear-end accidents more likely.

Table 1: Relationship between predictor variable and accident rate

Value of Exponent	Relationship with accident rate
$b_i > 1$	For increasing values of the variable, the number of accidents will increase, and at an increasing accident rate
$b_i = 1$	For increasing values of the variable, the number of accidents will increase at a constant (or linear) accident rate
$0 < b_i < 1$	For increasing values of the variable, the number of accidents will increase at a decreasing accident rate
$b_i = 0$	There will be no change in the number of accidents with increases in the value of the variable
$b_i < 0$	For increasing values of the variable, the number of accidents will decrease

In the case of two discrete variables where a feature is either present or not present, the resulting multipliers in the models are usually incorporated into the parameter b_0 for ease of use. In this case two b_0 values are presented, for both with and without the feature.

URBAN SIGNALS

The most recent work in New Zealand on developing accident prediction models for all reported vehicle accidents at a wide variety of urban and rural intersection types was undertaken by Turner (2000). This research included models for urban traffic signals. Models for total accident and major accident types were developed for signalised urban T-Junctions and cross-roads. Sites were selected throughout New Zealand. Five years of accident data were extracted from the LTSA's (now the Ministry of Transport's) accident database for each site and classified by movement type. Manual turning traffic volumes were also collected, typically for a 2-hour duration in the morning and evening peaks and the inter-peak.

The models used the annual average daily traffic from the mid-point of the five-year accident history. This was estimated from observed counts by applying daily, weekly, seasonal and annual correction factors. This was the case in studies that followed, including when using pedestrian and cyclist flow variables.

Signalised Cross Roads

The mean annual number of accidents at signalised cross-roads can be predicted by accident type and approach using the equations in Table 2 and the parameters in Table 3. Figure 2 illustrates the different conflicting and approach flows at cross-roads. The entering flow Q_e is the sum of all flows entering the intersection from each approach.

Table 2: Signalised cross-road accident prediction equations

Accident Type	Accident Codes	Equation (accidents per approach)
Crossing (No Turns)	HA	$A = b_0 \times q_2^{b_1} \times q_{11}^{b_2}$
Right Turn Against	LB	$A = b_0 \times q_2^{b_1} \times q_7^{b_2}$
Rear-end	FA to FE	$A = b_0 \times Q_e^{b_1}$
Loss-of-control	C & D	$A = b_0 \times Q_e^{b_1}$
Others		$A = b_0 \times Q_e^{b_1}$

Table 3: Signalised cross-roads – prediction model parameters

Accident Type	b_0	b_1	b_2	Error Structure
Crossing (No Turns)	1.57×10^{-4}	0.36	0.38	NB (K = 1.1)*
Right Turn Against	9.57×10^{-5}	0.49	0.42	NB (K = 1.9)*
Rear-end	1.66×10^{-6}	1.07	-	NB (K = 1.7)*
Loss-of –control	2.96×10^{-6}	0.95	-	NB (K = 0.8)*
Others	1.26×10^{-3}	0.46	-	NB (K = 1.5)*

*K is the Gamma shape parameter for the negative binomial (NB) distribution.

Excluding the model for rear-end accidents, all other models in Table 3 have exponents for vehicle flows between zero and one. This indicates that as traffic flows increase, the accident risk per vehicle decreases. As the exponent for entering flow in the rear-end accident model is above one, this indicates increasing accident risk per vehicle for increasing vehicle flows. This is an expected result for this accident type has been observed in an earlier study (Turner, 1995) and for other intersection types (Turner, 2000).

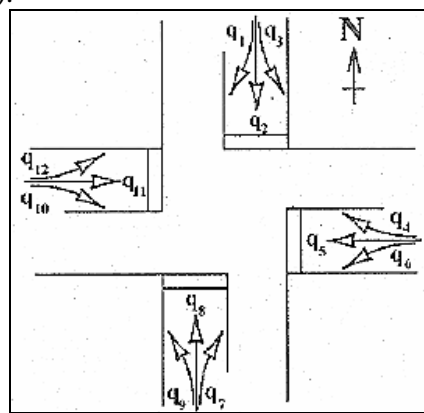


Figure 2: Conflicting and approach flow types (Cross-roads)

Signalised T-junctions

The mean annual number of accidents at signalised T-junctions are predicted by accident type and approach using the equations in Table 4 and the parameters in Table 5. Figure 3 illustrates the different conflicting and approach flows at T-junctions. The entering flow Q_e is the sum of all flows entering the intersection for each of the three approaches.

Table 4: Signalised T-junction accident prediction equations

Accident Type	Accident Codes	Equation (accidents per approach)
Right Turn Against	LB	$A = b_0 \times q_5^{b_1} \times q_3^{b_2}$
Rear-end	FA to FD	$A = b_0 \times Q_e^{b_1}$
Crossing (Vehicle Turning)	JA	$A = b_0 \times q_5^{b_1} \times q_1^{b_2}$
Loss-of –control	C & D	$A = b_0 \times Q_e^{b_1}$
Others		$A = b_0 \times Q_e^{b_1}$

Table 5: Signalised T-junction – prediction model parameters

Accident Type	b_0	b_1	b_2	Error Structure
Right Turn Against	1.08×10^{-1}	-0.38	0.56	Poisson
Rear-end	7.66×10^{-8}	1.45	-	NB (K = 0.5)
Crossing (Vehicle Turning)	2.67×10^{-2}	-0.30	0.49	NB (K = 1.2)
Loss-of –control	1.91×10^{-3}	0.17	-	Poisson
Others	1.69×10^{-2}	0.15	-	NB (K = 2.4)

The negative exponents in the ‘right-turn-against’ and ‘crossing’ (JA) models indicate that intersections with higher flows have fewer accidents. It is speculated that at high right turning flows the installation of right turn bays and exclusive right turn phases reduces accident occurrence. This is a matter considered in the latter study by Turner and Roozenburg (2004). It is not obvious why the exponent is negative for the ‘crossing’ (JA) model. However, it may be due to the relatively small number of accidents (12) and intersections (30). Further research is required to confirm this result.

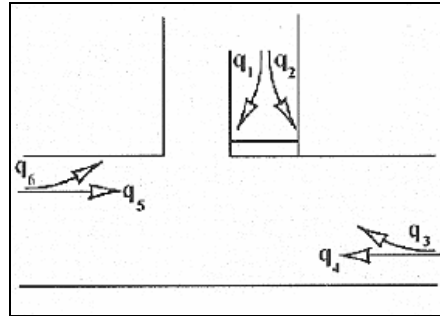


Figure 3: Conflicting and approach flow types (T-junction)

Product of Links

Following completion of the research report the ‘product-of-link’ models were further developed and incorporated into Transfund’s (now Land Transport NZ’s) Project Evaluation Manual. These models enable the accident rate (accidents per year) at an intersection to be predicted using the link (two-way) flows on intersecting roads.

The product of link cross-road model should not be used when the volumes of traffic on opposite arms of a four-arm intersection differ by more than 25% of the higher flow. This occurs when the majority of traffic on a link turns left or right, so that the opposing intersection arm has low traffic volumes. Where volumes on both opposing arms of a link are available then the two approach flows should be summed to calculate the link volume. Likewise, the model for T-junctions should not be used when the volumes of traffic differ by more than 25% on each of the opposing arms of the main road. This occurs at intersections where the majority of traffic travels to or from one of the main road approaches to the stem of the T.

The total reported annual number of injury accidents for each intersection types is determined using the following equations:

Equation 5
$$A_T = b_0 \times Q_{minor}^{b1} \times Q_{major}^{b2}$$

Equation 6
$$A_T = b_0 \times Q_{stem}^{b1} \times Q_{major}^{b2}$$

Where Q_{minor} is the lowest of the two-way link volumes for cross-roads, and Q_{stem} the stem flow for T-junctions.

Table 6: ‘Product-of-link’ flow model parameters for signalised intersections

Accident Type	b_0	b_1	b_2	Error Structure
Signalised cross-road	3.69×10^{-3}	0.14	0.46	NB (K = 4.8)
Signalised T-junction	1.73×10^{-1}	0.12	0.04	NB (K = 4.6)

NON-FLOW VARIABLES

Turner (2000) identified that the daily volume of right turning and through traffic are key predictor variables for accident occurrence in right-turn against accidents at 4-arm, two-way traffic signals. Turner and Roozenburg (2004) set out to consider a number of non-flow variables, and determine whether these are also key predictors for accident occurrence. The outcome is a more refined accident prediction model for this accident type. Of particular interest was the impact of right turn phasing on accident occurrence.

The key non-flow variables examined were; right-turn signal phasing, visibility to opposing traffic, number of opposing lanes, right turn bay offset and intersection depth. A number of the variables were correlated and hence explain the same variability in the accident data.

To investigate which single variables should be added to the model we considered the change in the log-likelihood function for each model. Table 7, shows the log-likelihood for each model when the non-flow variable was added to the two flow variables. The number of lanes of opposing through traffic maximises the log-likelihood function, which indicates that it is the best variable to add to the model. Interestingly visibility to opposing through traffic does not feature as an important predictor variable.

Table 7: Log-likelihood and BIC comparison

Model	Log-likelihood	BIC
Conflicting flow only model (flow-only model)	-384.75	1.732
Visibility to opposing traffic	-384.58	1.744
Right turn visibility below standard	-383.61	1.740
Number of lanes of opposing through traffic	-378.87	1.719
Right-turn signal phasing	-384.63	1.744
Opposing right turning flow	-383.86	1.741
Intersection depth	-383.84	1.741

Using the Bayesian Information Criterion (BIC) we can compare the model that includes the number of lanes with the flow only model. This BIC statistic indicates that the new model, which includes the number of opposing lanes, is a better model than that the flow only model. We considered whether any of the other predictor variables should be added to the new three variable model but the BIC statistic does not indicate that this would produce a better model. The analysis indicates that the preferred model form for right-turn against accidents is:

$$\text{Equation 6} \quad A_T = b_{0i} \times q_2^{0.44} \times q_7^{0.39}$$

where:

$$b_{0i} = 1.05 \times 10^{-4} \quad \text{for an approach with a single opposing through lane}$$

$$b_{0e} = 2.06 \times 10^{-4} \quad \text{for an approach with multiple opposing lanes}$$

Two factors that did not improve the flow only model were signal phasing and visibility. In the case of the model for signal phasing, sites were classed as either having fully filtered right turn phases or partially and fully protected right turn phases. The partially and fully protected signals phasing sites, had to be grouped together because only a few sites in the dataset had fully protected right turn phases. This may be the reason why the log-likelihood did not decrease significantly, as these two site types may have different effects, and this may be hidden when they are combined. It is less clear as to why visibility did not feature in the modelling. One hypothesis is that where visibility is limited drivers may be more cautious.

A possible extension of this research would be to sample more intersections with fully controlled right turn phases. This would enable a more thorough investigation of the effect of right-turn signal phasing.

Further investigation is also required to establish what driver behaviours are influencing right turn against accidents. Issues that need to be examined;

- 1) Are drivers more cautious when visibility to through vehicles is restricted, due to right turn bay offset and short intersection depth; and
- 2) Are drivers more cautious when right turns are filtered, and are they less likely to run the red-light than when right turn phases are provided.

Modeling of driver behaviour when turning right at traffic signals in a driver simulator, would be useful.

PEDESTRIAN AND CYCLISTS VERSUS MOTOR VEHICLE MODELS

Turner et. al. (2005) developed APMs for pedestrians and cyclists. Prior to this research, APMs had been developed using only motor vehicle volumes. Turner et. al (2005) used a generalised linear modelling technique to develop APMs for cyclists versus motor-vehicles at roundabouts, signalised cross-roads and midblock. Pedestrian versus motor-vehicle APMs were developed for signalised cross-roads and midblock.

Figure 4 compares the proportions of accidents involving pedestrians and cyclists over the New Zealand urban network. Accidents involving cyclists make up 7 % of all injury accidents at signalised cross-roads in urban areas. Pedestrian accidents make up 18 %.

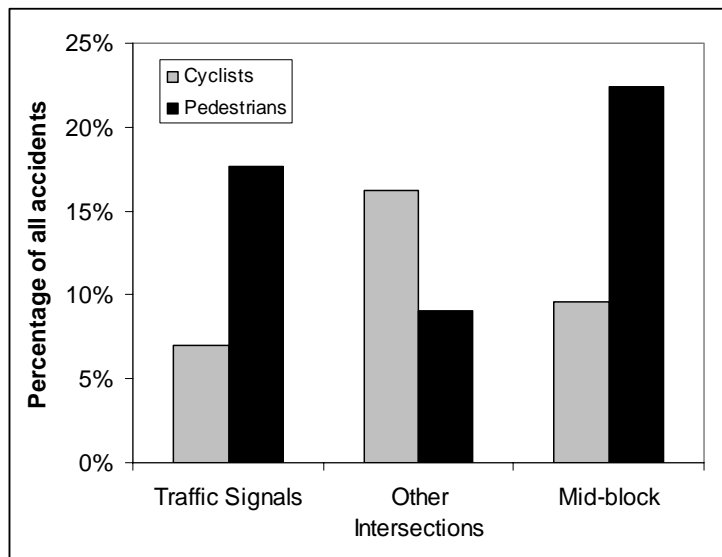


Figure 4: Proportion of injury accidents involving pedestrians and cyclists (2000-2004)

Two model types were developed for cycle versus motor-vehicle accidents at signalised cross-roads. The first, a ‘same direction’ model predicts accidents on a single approach between cyclists either colliding with a stationary vehicle or moving motor-vehicle, travelling in the same direction. The second model is for right-turn-against accidents where a cyclist is travelling straight through the intersection and collides with a motor vehicle turning right. Table 8 and Table 9 present these

two models and the proportion of cycle accidents that they represent at signalised cross-roads. Cycle movements are coded in a similar manner to motor-vehicle movements (see Figure 2). Entering flows, for example C_e , are the sum of all cycle entering flows, for example $c_1 + c_2 + c_3$. Figure 5 shows the movements graphically.

Table 8: Signalised cross-road cycle accident prediction equations

Accident Type	Accident Codes	Equation (accidents per approach)	Proportion of Cycle Accidents
Same Direction	A, E, F, G	$A = b_o \times Q_e^{b_1} \times C_e^{b_2}$	35%
Right Turn Against – Motor vehicle turning	LB	$A = b_o \times q_7^{b_1} \times c_2^{b_2}$	21%

Table 9: Signalised cross-roads – cycle accident prediction model parameters

Accident Type	b_0	b_1	b_2	Error Structure
Same Direction	7.49×10^{-4}	0.29	0.09	Poisson
Right Turn Against – Motor vehicle turning	4.41×10^{-4}	0.34	0.20	NB, K=1.3

*K is the Gamma shape parameter for the negative binomial distribution.

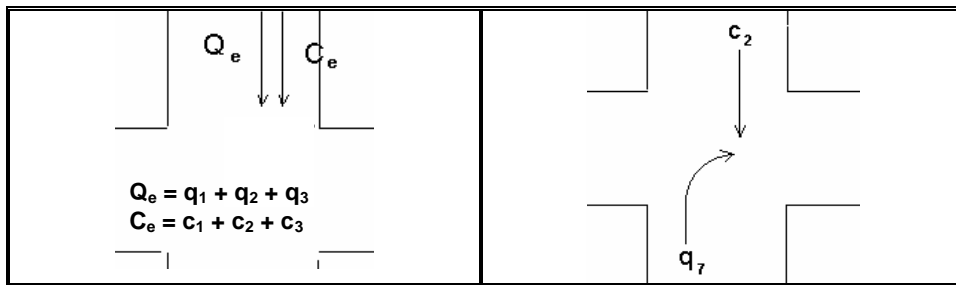


Figure 5: Cycle model variables

The small value of the exponent of cycle flows (b_2) in Table 9 indicates a ‘safety in numbers’ effect where the accident rate per cyclist decreases substantially as the number of cyclists increases. Figure 6 illustrates this effect for ‘same direction’ cycle accidents. This effect has also been identified in international studies (refer to Turner et.al, 2005).

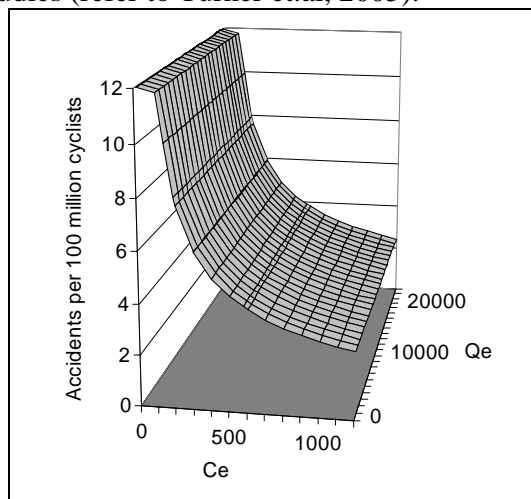


Figure 6: Accident rate for ‘same direction’ cycle accidents

Two models were developed for predicting pedestrian accidents at signalised cross-roads. The majority of all accidents involving pedestrians, not just those at signalised cross-roads, occur where

vehicles are travelling straight along the road and the pedestrian is crossing. These accidents represent 50% of pedestrian accidents at signalised cross-roads. The second major type of pedestrian accidents at signalised cross-roads is where right turning vehicles collide with pedestrians crossing the side road. Table 10 and Table 11 present these two models. Pedestrian movements differ from motor-vehicle and cycle movements. For cross-roads, four movements are used, one for pedestrians crossing each approach. One model presented here also uses a different motor vehicle movement, Q , which is the two-way vehicle flow on one road. Figure 7 shows the movements used graphically.

Table 10: Signalised cross-road pedestrian accident prediction equations

Accident Type	Accident Codes	Equation (accidents per approach)	Proportion of Ped. Accidents
Crossing – vehicle intersecting	NA, NB	$A = b_o \times Q^{b_1} \times P^{b_2}$	50%
Crossing – vehicle turning right	ND, NF	$A = b_o \times q_4^{b_1} \times p_1^{b_2}$	36%

Table 11: Signalised cross-roads – pedestrian accident prediction model parameters

Accident Type	b_0	b_1	b_2	Error Structure
Crossing – vehicle intersecting	7.28×10^{-6}	0.63	0.40	NB, K=3.7*
Crossing – vehicle turning right	5.43×10^{-5}	0.43	0.51	NB, K=0.7*

*K is the Gamma shape parameter for the negative binomial (NB) distribution.

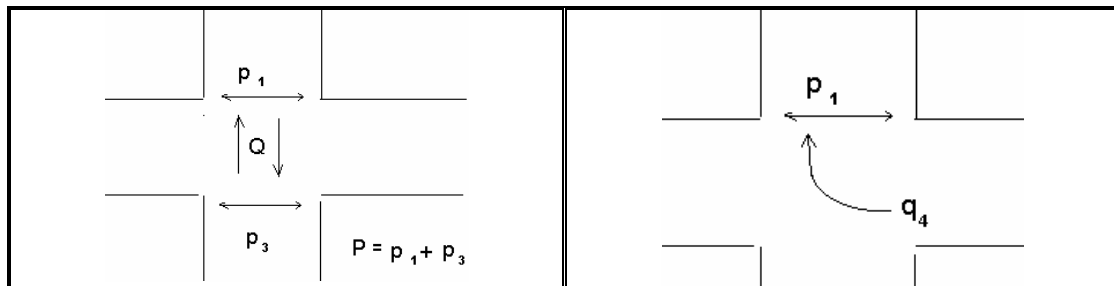


Figure 7: Pedestrian model variables

Unlike cycle APMs, the ‘safety in numbers effect’ is not as pronounced with exponents of flow being similar to those observed for motor vehicle flows.

Using the models below, it is possible to predict the total number of pedestrian and cycle accidents at signalised cross-roads. This is done by calculating the number of accidents on each approach using the models and then multiplying the result by factors to take into account ‘other’ types of accidents. The models generally have the form:

Equation 7 $A_{T(cycle)} = factor \times (A_{Cycle Type 1} + A_{Cycle Type 2});$ and

Equation 8 $A_{T(pedestrian)} = factor \times (A_{Pedestrian Type 1} + A_{Pedestrian Type 2})$

HIGH SPEED INTERSECTION MODELS

Beca are currently developing accident prediction models for rural intersections (or intersections with speed limits above 80 km/h). Rural intersections are particularly hazardous where there are high operating speeds on one or more approaches. The first stage of this project, developing

accident prediction models for rural priority junctions (cross-roads and T-junctions) is now complete. The second stage of this project involves developing accident prediction models for rural traffic signals and roundabouts.

Experience suggests that traffic signals in high-speed areas have high accident rates and more severe accidents than those in lower speed areas. However, this has not yet been quantified. This study is required to evaluate the relative accident rates and costs of traffic signals in rural or high-speed areas compared with lower speed areas. The models can also be used to compare alternative options, such as roundabouts, over-bridges (in which access is severed) and grade separated interchanges in high-speed areas.

The number of high speed and rural traffic signal sites (where at least two legs of the intersection are 80km/h or higher) in New Zealand is relatively low (approximately 20 sites). To develop good fitting models it is desirable to have large sample sets (50 plus intersections). Given such a sample size was not available in New Zealand, two alternatives were investigated:

1. To combine both urban and rural traffic signals datasets and develop covariate models based on speed limit for each accident type and for total accidents. In this technique the full dataset is used to develop the 'exponents' for the model (b_1 and b_2) and the dataset for each speed limit would be used to calculate the multiplicative parameter (b_0).
2. To include traffic signal data from Melbourne in Australia and produce models based on 31 sites in Melbourne and around 20 sites in New Zealand. A covariate analysis could again be used to assess the differences in accident occurrence in the two countries.

The second option was selected, due to availability of data in Melbourne. It is also a good opportunity to compare the accident rates at intersections in both countries at the site-level, rather than by state, as is normally the case. Traffic count data was collected in both New Zealand and Melbourne from SCATS controllers and in the case of shared lanes, from manual counts.

CONCLUSIONS

This paper has presented and discussed accident prediction models that have been developed for signalised intersections in New Zealand. Flow-only models are now available for:

- Total reported accidents (involving a motor-vehicle);
- Cyclist versus motor vehicle accidents;
- Pedestrians versus motor vehicle accidents; and
- Major accident types;

How to interpret the model forms has been presented along with the basic model structures, both in the multiplicative and linear forms. The model relationships developed provide us with knowledge of the accident causing mechanisms in accidents at traffic signals. It has been shown that the key variables at traffic signals are motor-vehicle, cyclist and pedestrian flows and that there is generally a decrease in the accident rate per vehicle, pedestrian and cyclist with increasing flows. This is the 'safety in numbers effect'. The biggest effect is for cyclists.

The effect of non-flow variables on right-turn-against accidents at traffic signals have also been discussed. The most important non-flow predictor variable is the number of opposing lanes. The addition of visibility and right-turn signal phasing predictor variables were shown not to improve the flow only model, however, further research is required to confirm this.

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