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Isomorphism in Kinematic Chains Using the Path Matrix

1. ABSTRACT

This paper presents a new method to identify isomorphism in kinematic chains (KC). The method is based on a theoretic approach and path matrix (PM) properties of KCs. In comparison with the 'adjacent' matrix, the PM is simpler, much easier to visualise, and more directly describes the topological relationship among the KCs. Using the PM, the isomorphism among KCs can be easily determined. Among the existing methods for identifying isomorphism among KCs and co-spectral graphs, the PM method is the least calculation-intensive. Two new composite structural invariants – the sum of absolute characteristic polynomial coefficients (**PMP∑**) and the maximum absolute value of the characteristic polynomial coefficient (**PMPmax**) – have been derived from the PM.

This study will help the designer to select the best KC to perform the specified task at the conceptual stage of design. The proposed method is explained with examples.

Key words: path matrix, kinematic chains, structural invariants, isomorphism

2. INTRODUCTION

Structural analysis and synthesis is very important in the design of mechanisms. Identifying isomorphism among KCs is an essential step in kinematic mechanism synthesis. Undetected isomorphism results in duplicate solutions and unnecessary effort, and falsely identified isomorphism eliminates possible candidates for new mechanisms. Identifying isomorphisms of kinematic chains using the characteristic polynomial method is simple (Yan & Hall 1981), but the reliability of this method is in question, as several apparently incorrect results were found (Mruthyunjaya 1987). Some new approaches to these problems, such as eigenvector (Chang et al. 2002), incident degree (Jongsma et al. 1992, Jensen 1992), group theory (Tuttle et al. 1989), adjacent-chain table (Chu & Cao 1992) and artificial neural-network (Kong, Li & Zhang 1999) methods were also investigated. Most of these latter methods are based on an adjacency matrix (Uicher & Raicu 1975) or a distance matrix (Rao 1988), however these methods are complex, difficult to apply and fail to reliably detect uniqueness or take too much time for determining isomorphism of a kinematic chain.

This paper presents a new method to identify isomorphism among KCs, by comparing structural invariants PMP∑ and PMPmax of the PM of the KC. The method suggested in this paper is applicable to all planar KCs having simple joints.

3. DEFINITION OF TERMINOLOGY

A number of new terms have been developed to describe a KC. The links of a chain are labelled with positive integers 1, 2, 3, etc while the letters a, b, c, etc label the joints of a KC. Links and joints of a KC are arbitrarily labelled.

3.1 Link Path

Link path is defined (Yadav, Pratap & Agarwal 1996) as the alternating sequence of distinct links and distinct joints, starting and ending with the links, such that each joint connects the links preceding and following it. The length of a link path is defined as the number of joints in the path. For example, the sequence 1, a, 5, b, 3, c, 10 in the KC shown in Figure 1, is a link path with length 3 (ie three joints – a, b and c).

3.2 Shortest Length of a Link Path

The shortest length of a link path is defined as the minimum sum of all simple joints in a link path. For example, the shortest length of a link path 1, a, 5, b, 3, c, 10 in the KC shown in Figure 1 is 3.

3.3 Path Matrix

For an n-link KC, the PM is defined (a similar matrix has been used as flow matrix (Rao 1989)) as an n x n symmetric matrix whose i^{th}, j^{th} element, P_{ij} is defined as the shortest length of a link path between link i and j if link i and j are connected directly or indirectly to each other. If i=j then $P_{ij}=0$.

4. STRUCTURAL INVARIANTS PMP∑ AND PMPmax

Polynomial coefficient values (**PMP**) are the characteristic invariants for KC. Many investigators report co-spectral graphs (non-isomorphic KC with the same eigenvalues). But these eigen-spectra (eigenvalues or polynomial coefficient values) have been determined from (0,1) adjacency matrices. The proposed PM provides a different set of characteristic polynomials of the co-spectral graph. To make this PM polynomial spectrum a powerful single number characteristic index, new composite structural invariants are proposed. These indices are PMP∑ and PMPmax of the PM. The polynomial coefficient values of the PM are obtained using standard MATLAB software.
D (λ) gives the characteristic polynomial of the PM. The monic-polynomial of degree n of the PM is given by:

$$\begin{aligned} D(\lambda) &= \left| PM - \lambda I \right| \\ &= \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots + a_{n-1}\lambda + a_n \end{aligned}$$

Here n = number of links and 1, a_1, $a_2 \ldots a_{n-1}$, a_n are characteristic polynomial coefficients.

The two important properties of the characteristic polynomials are:

(1) The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is an invariant.

$$1 + |a_1| + |a_2| + \ldots + |a_{n-1}| + |a_n| = \text{invariant}$$

(2) The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is also invariant for a PM. Therefore, two structural invariants are proposed – one is PMP∑ and the other is PMPmax.
These invariants are unique and can be used as an identification number to detect the isomorphism among KCs.

5. PROPOSED TEST – BASIS

KCs are complex chains with combinations of binary, ternary and other higher-order links. These links are joined by simple joints. The KC is the assembly of link/pair combinations. While considering structural equivalence, it is essential to consider the number of links/joints and layout of the links in the assembly. An identification number is assigned to each joint. A simple joint has a value of one; two simple joints a value of two; three simple joints a value of three and so on. Joints values are used to assign values to the PM and these are used to identify the layout of the KC. For determining isomorphism in a KC, the PM is determined and PMP∑ and PMPmax are compared with the respective KCs.

6. IDENTIFYING KC ISOMORPHISM

Let the two KCs be represented by the two similar matrices A and B such that $B = P^{-1}AP$, taking into account that the matrix λI commutes with the matrix P and $|P^{-1}| = |P|^{-1}$. We have:

$$\begin{aligned} \left| B - \lambda I \right| &= \left| P^{-1}AP - \lambda I \right| \\ &= \left| P^{-1}(A - \lambda I)P \right| \\ &= \left| P^{-1} \right| \cdot \left| (A - \lambda I) \right| \cdot \left| P \right| \\ &= \left| A - \lambda I \right| \end{aligned}$$

Theory states: “Similar matrices have the same characteristic polynomials and same characteristic roots (eigenvalues)”.

This means that if the characteristic polynomial coefficients of two PM representing two KC are the same, their structural invariants PMP∑ and PMPmax will be the same and the two KCs can be considered isomorphic (Hildebrand 1965).

7. ILLUSTRATIVE EXAMPLES

7.1 Example 1

The first example concerns three KCs with 12 bars, and a single degree-of-freedom as shown in Figures 1, 2 and 3. The task is to determine whether these three KCs are isomorphic. The PM for these KCs are represented by PM1, PM2 and PM3 respectively.

PM1 =

Link	1	2	3	4	5	6	7	8	9	10	11	12
1	0	2	2	3	1	1	2	2	3	3	3	4
2	2	0	2	3	1	3	2	4	1	3	3	2
3	2	2	0	4	1	3	4	2	3	1	3	2
4	3	3	4	0	4	2	1	2	2	3	1	2
5	1	1	1	4	0	2	3	3	2	2	4	3
6	1	3	3	2	2	0	1	1	2	2	2	3
7	2	2	4	1	3	1	0	2	1	3	2	2
8	2	4	2	2	3	1	2	0	3	1	1	2
9	3	1	3	2	2	2	1	3	0	2	2	1
10	3	3	1	3	2	2	3	1	2	0	2	1
11	3	3	3	1	4	2	2	1	2	2	0	1
12	4	2	2	2	3	3	2	2	1	1	1	0

PM2 =

Link	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	2	3	2	1	1	2	3	3	2	3
2	1	0	1	2	3	2	2	1	2	2	3	4
3	2	1	0	1	2	3	3	2	2	1	2	3
4	3	2	1	0	1	2	4	3	3	2	3	2
5	2	3	2	1	0	1	3	4	4	3	2	1
6	1	2	3	2	1	0	2	3	4	4	3	2
7	1	2	3	4	3	2	0	1	2	2	1	2
8	2	1	2	3	4	3	1	0	1	2	2	3
9	3	2	2	3	4	4	2	1	0	1	2	3
10	3	2	1	2	3	4	2	2	1	0	1	2
11	2	3	2	3	2	3	1	2	2	1	0	1
12	3	4	3	2	1	2	2	3	3	2	1	0

PM3 =

Link	1	2	3	4	5	6	7	8	9	10	11	12
1	0	2	3	2	1	2	3	2	4	3	1	3
2	2	0	3	2	1	2	3	4	2	3	3	1
3	3	3	0	4	2	1	2	2	2	1	3	3
4	2	2	4	0	3	4	3	2	2	3	1	1
5	1	1	2	3	0	1	2	3	3	4	2	2
6	2	2	1	4	1	0	1	2	2	3	3	3
7	3	3	2	3	2	1	0	1	1	2	2	2
8	2	4	2	2	3	2	1	0	2	1	1	3
9	4	2	2	2	3	2	1	2	0	1	3	1
10	3	3	1	3	4	3	2	1	1	0	2	2
11	1	3	3	1	2	3	2	1	3	2	0	2
12	3	1	3	1	2	3	2	3	1	2	2	0

For KC1 (Figure 1), the characteristic polynomial coefficients (PMP) are: 1.0e+004 * [0.0001, 0.0000, -0.0378, -0.4498, -2.0144, -3.6302, -2.0439, 0.3852, 0.4212, 0.0000, -0.0000, -0.0000, -0.0000]

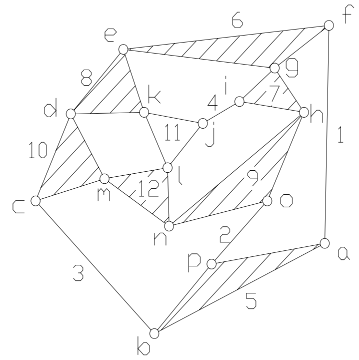

Figure 1: Twelve bar chain, single freedom

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 8.9826e+004. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 3.6302e+004.

For KC2 (Figure 2), the characteristic polynomial coefficients (PMP) are: 1.0e+004 * [0.0001, -0.0000, -0.0378, -0.4498, -2.0144, -3.6302, -2.0439, 0.3852, 0.4212, -0.0000, 0.0000, -0.0000, 0.0000]

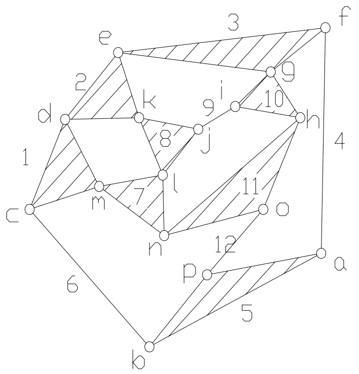

Figure 2: Twelve bar chain, single freedom

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 8.9826e+004. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 3.6302e+004.

For KC3 (Figure 3) the characteristic polynomial coefficients (PMP) are: 1.0e+004 * [0.0001, 0.0000, -0.0376, -0.4510, -2.0063, -3.1668, 0.4980, 3.9936, 1.4208, -0.4608, -0.0000, 0.0000, 0.0000]

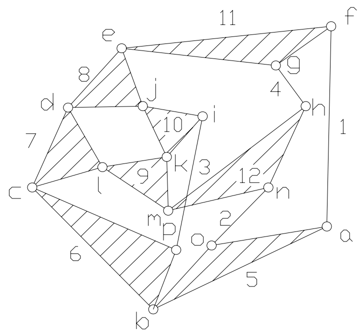

Figure 3: Twelve bar chain, single freedom

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 1.2035e+005. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 3.9936e+004.

Our method reports that KC1 and KC2 are isomorphic as the corresponding values of PMP∑ and PMPmax are the same for KC1 and KC2 respectively. Similarly, our method reports that KC1 and KC3 are non-isomorphic as the values of PMP∑ and PMPmax are different for KC1 and KC3 respectively. Note that the same conclusion can be obtained using other methods – eigenvector (Chang et al. 2002) and artificial neural-networks (Kong, Li & Zhang 1999).

7.2 Example 2 (Multi Degree-of-freedom KCs)

The second example concerns two KCs with 10 bars, 12 joints and three degrees-of-freedom as shown in Figures 4 and 5.

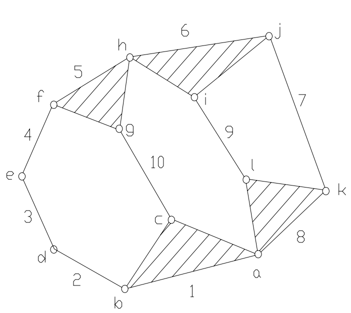

Figure 4: Ten bar chain, three freedom

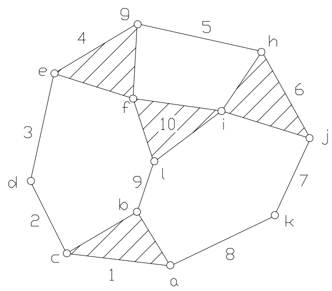

Figure 5: Ten bar chain, three freedom

The task is to examine whether these two KCs are isomorphic. The PM for these KCs are represented by PM4 and PM5 respectively.

PM4 =

Link	1	2	3	4	5	6	7	8	9	10
1	0	1	2	3	2	3	2	1	2	1
2	1	0	1	2	3	4	3	2	3	2
3	2	1	0	1	2	3	4	3	4	3
4	3	2	1	0	1	2	3	4	3	2
5	2	3	2	1	0	1	2	3	2	1
6	3	4	3	2	1	0	1	2	1	2
7	2	3	4	3	2	1	0	1	2	3
8	1	2	3	4	3	2	1	0	1	2
9	2	3	4	3	2	1	2	1	0	3
10	1	2	3	2	1	2	3	2	3	0

PM5 =

Link	1	2	3	4	5	6	7	8	9	10
1	0	1	2	3	4	3	2	1	1	2
2	1	0	1	2	3	4	3	2	2	3
3	2	1	0	1	2	3	4	3	3	2
4	3	2	1	0	1	2	3	4	2	1
5	4	3	2	1	0	1	2	3	3	2
6	3	4	3	2	1	0	1	2	2	1
7	2	3	4	3	2	1	0	1	3	2
8	1	2	3	4	3	2	1	0	2	3
9	1	2	3	2	3	2	3	2	0	1
10	2	3	2	1	2	1	2	3	1	0

For KC4 (Figure 4), the characteristic polynomial coefficients (PMP) are: 1.0e+004 * [0.0001 0.0000 -0.0257 -0.2400 -0.7928 -0.8384 0.5296 1.4080 0.6144 -0.0000 -0.0000].

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 4.4490e+004. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 1.4080e+004.

For KC5 (Figure 5), the characteristic polynomial coefficients (PMP) are: 1.0e+003 * [0.0010, 0, -0.2570, -2.3600, -7.3120, -6.5920, 2.4960, 4.0960, -0.0000, 0.0000, 0.0000].

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 2.3114e+004. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 7.3120e+003.

Our method reports that KC4 and KC5 are non-isomorphic as the values of PMP∑ and PMPmax are different for both KCs. Note that the same conclusion is obtained using summation polynomials (Shende & Rao 1994).

7.3 Example 3

The third example concerns another two KCs with 10 bars, 13 joints, and a single degree-of-freedom as shown in Figures 6 and 7.

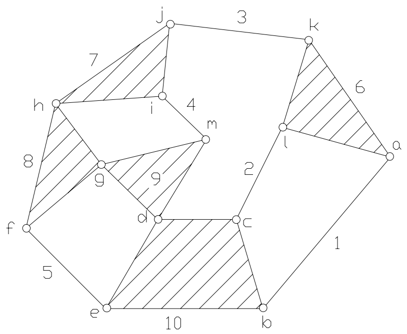

Figure 6: Ten bar chain, single freedom

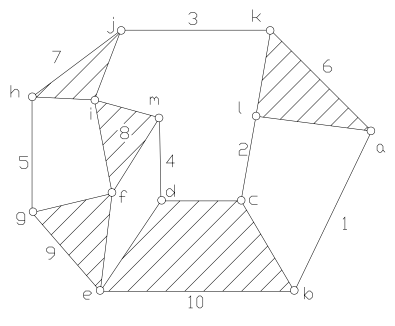

Figure 7: Ten bar chain, single freedom

The task is to examine whether these two KCs are isomorphic. The PM matrices for these KCs are represented by PM6 and PM7 respectively.

PM6 =

Link	1	2	3	4	5	6	7	8	9	10
1	0	2	2	4	2	1	3	3	3	1
2	2	0	2	3	3	1	3	3	2	1
3	2	2	0	2	3	1	1	2	3	3
4	4	3	2	0	3	3	1	2	1	2
5	2	3	3	3	0	3	2	1	2	1
6	1	1	1	3	3	0	2	3	3	2
7	3	3	1	1	2	2	0	1	2	3
8	3	3	2	2	1	3	1	0	1	2
9	3	2	3	1	2	3	2	1	0	1
10	1	1	3	2	1	2	3	2	1	0

PM7 =

Link	1	2	3	4	5	6	7	8	9	10
1	0	2	2	3	3	1	3	3	2	1
2	2	0	2	2	4	1	3	3	3	1
3	2	2	0	3	2	1	1	2	3	3
4	3	2	3	0	3	3	2	1	2	1
5	3	4	2	3	0	3	1	2	1	2
6	1	1	1	3	3	0	2	3	3	2
7	3	3	1	2	1	2	0	1	2	3
8	3	3	2	1	2	3	1	0	1	2
9	2	3	3	2	1	3	2	1	0	1
10	1	1	3	1	2	2	3	2	1	0

For KC6 (Figure 6), the characteristic polynomial coefficients (PMP) are: 1.0e+003 * [0.0010, -0.0000, -0.2330, -2.1280, -7.1850, -9.5360, -3.1850, 2.1580, 1.1650, -0.0000, -0.0090].

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 2.5600e+004. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 9.5360e+003.

For KC7 (Figure 7), the characteristic polynomial coefficients (PMP) are: 1.0e+003 * [0.0010, 0.0000, -0.2330, -2.1280, -7.1850, -9.5360, -3.1850, 2.1580, 1.1650, 0.0000, -0.0090].

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 2.5600e+004. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 9.5360e+003.

Our method reports that KC6 and KC7 are isomorphic as the set of values for PMP∑ and

PMPmax are the same for both the KCs. Note that the same conclusion is obtained using artificial neural-networks (Kong, Li & Zhang 1999).

7.4 Example 4

The fourth example concerns two KCs with four bars, four joints, and a single degree-of-freedom as shown in Figures 8 and 9.

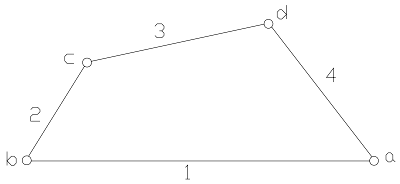

Figure 8: Four bar chain, open configuration

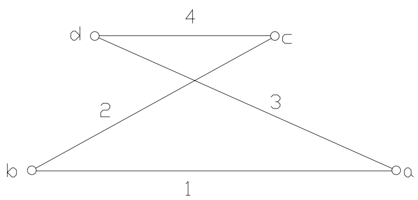

Figure 9: Four bar chain, crossed configuration

The task is to examine whether these two KCs are isomorphic. Following the same procedure:

For KC8 (Figure 8), the characteristic polynomial coefficients (PMP) are: [1.0000, -0.0000, -12.0000, -16.0000, -0.0000].

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 29.0000. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 16.0000.

For KC9 (Figure 9), the characteristic polynomial coefficients (PMP) are: [1.0000, -0.0000, -12.0000, -16.0000, -0.0000].

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 29.0000. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 16.0000.

Our method reports that KC8 and KC9 are isomorphic as the values for PMP∑ and PMPmax are same for both the KCs.

7.5 Example 5

The fifth example concerns two KCs with six bars, seven joints, a single degree-of-freedom as shown in Figures 10 and 11.

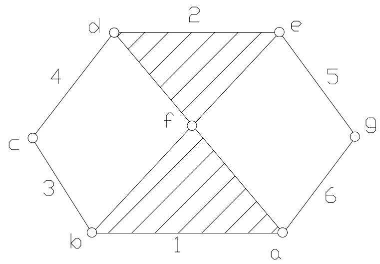

Figure 10: Watt six-bar chain

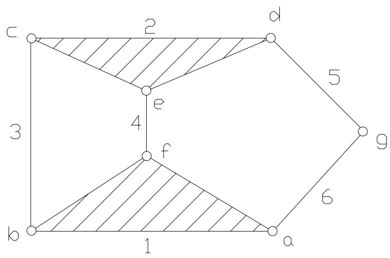

Figure 11: Stephenson six-bar chain

The task is to examine whether these two KCs are isomorphic.

For KC10 (Figure 10), the characteristic polynomial coefficients (PMP) are: [1.0000, 0.0000, -49.0000, -168.0000, -144.0000 0.0000, -0.0000].

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is 362.0000. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 168.0000.

For KC11 (Figure 11), the characteristic polynomial coefficients (PMP) are: [1.0000, 0.0000, -39.0000, -136.0000, -147.0000, -28.0000, 4.0000].

The sum of the absolute values of the characteristic polynomial coefficients (PMP∑) is

355.0000. The maximum absolute value of the characteristic polynomial coefficient (PMPmax) is 147.0000.

Our method reports that KC10 and KC11 are non-isomorphic as the values of PMP∑ and PMPmax are different for both the KCs. Note that the same conclusion is obtained by using artificial neural-networks (Kong, Li & Zhang 1999).

8. RESULTS

The proposed invariants PMP∑ and PMPmax are able to detect isomorphism among KCs and even KCs with co-spectral graphs. All the 16 KCs of eight links, a single degree-of-freedom and 230 KCs of 10-links, a single degree-of-freedom have been tested for their non-isomorphism and no counter example has been detected.

9. CONCLUSION

This paper proposes a simple, efficient and reliable method to identify isomorphism among KCs. It incorporates all the features of the KC and as such, violation of the isomorphism test is difficult. This method uses the characteristic polynomials and composite structural invariants PMP∑ and PMPmax of the PM matrix of the KCs. The advantage is that they are very easy to compute using MATLAB software. It is not essential to determine both the composite invariants to compare two KCs – when the PMP∑ is the same then it determines PMPmax for both KCs. The PM matrices can be written with very little effort, even by mere inspection of the KC.

The proposed test is quite general in nature and can detect isomorphism of not only planar KCs of one degree-of-freedom, but also KCs of multiple degrees-of-freedom. The characteristic polynomials and composite structural invariants are very informative and from them valuable information regarding topology of KCs can be predicted. According to this method, we can find that the PM is a map of KCs and characteristic polynomials and other characteristic invariants may reflect some nature and inherent property of the mechanism. The inherent relationship between characteristic polynomial values and KCs needs further study.

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